

Intrinsic dissipation in cantilevers

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Abstract

We consider the effects of a velocity-independent friction force on cantilever damping. It is shown that this dissipation mechanism causes nonlinear effects in the cantilever vibrations. The size of the nonlinearity increases with decreasing cantilever velocity. Our analysis makes it possible to understand Stipe's *et al.* [1] experiments where an amplitude dependence of the cantilever eigenfrequency and anomalous dissipation was observed only at small amplitudes.

1 Introduction

The quality and potential practical opportunities for micromechanical devices strongly depend on their dissipative characteristics. Numerous authors have studied the damping of cantilever vibrations both theoretically and experimentally. There are at least two obvious motivations for these studies. First, the amount of dissipation determines the sensitivity of cantilever-based

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devices. Hence, reducing dissipation improves the technical characteristics of the devices. The second is based on the possibility of probing the dissipative forces which arise due to cantilever-sample interaction in order to obtain a useful information about the sample. The last idea was a basic one for originating a new direction for atomic force microscopy: noncontact dissipation force microscopy. (See references [2]-[7]).

Many attempts to understand the dissipation mechanisms in cantilevers were undertaken. The corresponding theoretical models, which could describe cantilever damping, are presented in references [8] and [9]. At the same time, the fundamental nature of damping is still not clear, especially in the case of noncontact cantilever vibrations. (See, for example, discussions in references [1],[2],[7], and [13]). Fluctuations of the van der Waals force (“vacuum friction”) or Joule losses are not responsible for the dissipation measured experimentally. The corresponding friction effects have been calculated in [10]-[13] to be many orders of magnitude smaller than those measured in [1]. There is not only quantitative disagreement, but also different qualitative behavior. The point is that both theories predict very strong dependence of damping on the cantilever-sample separation, and this is not consistent with the experimental observations.

A more complex and more specific model, involving adsorbed particles, was suggested in [14]. The authors connect the large long-range noncontact friction with the electromagnetic interaction of moving charges, induced on the surface of the tip by the bias voltage, with acoustic vibrations in an adsorbate layer on the surface of the sample.

An internal quantum friction due to the presence of a finite number of two-level systems is analyzed in Refs. [15] and [16]. Their analysis is related to millikelvin temperatures which is beyond experimental conditions of [1].

The experimental dependence of the friction force, F , on the bias voltage, V , applied between the tip and a sample, has been found. It is the quadratic dependence, $F \sim V^2$ (see, for example, references [1] and [7]) that is really typical of the Joule mechanism for energy losses. At the same time, it should be emphasized that not only Joule losses, but also a tip-sample attractive forces, caused by electrostatic interactions between closely spaced charged surfaces, behave as V^2 . Taking into account the fact that the Joule mechanism is not effective (at least for good conductors), the increased dissipation may be attributed to some effects of the attractive forces. In this connection, it should be noted that the increase of dissipation with decreasing tip-sample spacing was also obtained in the absence of the electrostatic attraction (for

zero-value bias-voltage). The calculations of [13] show that at small separations the van der Waals (Casimir) attractive force may become so strong as to be able to modify the cantilever eigenfrequencies.

A very specific attraction of the metal tip to dielectric samples occurs as well. Although overall the dielectric sample is electrically neutral, it may contain localized electric charges, which interact with the corresponding “image” charges in the tip. Samples of fused silica were used by authors of [1] to study the effect of charged centers on the cantilever vibrations. (Reference [13] contains the corresponding theoretical calculations.) The number of centers has been varied by means of irradiation with γ rays. An enhancement of dissipation occurs when the concentration of charged defects increases.

The increasing attractive forces in the above systems are accompanied by the growth of dissipation. Although this force may be of a different physical nature, it always results in an increase in cantilever dissipation. Therefore any consistent theory of dissipation should take into account this circumstance.

Concluding the introduction, we highlight a statistical analysis of various micromechanical systems [9] that shows that the quality factor (inverse of dissipation) usually scales linearly with size. There is a general tendency of dissipation to increase with the surface/volume ratio, indicating the relevance of the cantilever surface layers to dissipation.

2 The friction force

In general, the cantilever motion is described by the oscillator model. The corresponding equation of motion is given by

$$m \frac{\partial^2 X}{\partial t^2} + \Gamma \frac{\partial X}{\partial t} + kX = 0, \quad (1)$$

where $X = X(t)$ is the displacement of the cantilever tip; m and k are the cantilever effective mass and spring constants, respectively. The term $\Gamma \frac{\partial X}{\partial t}$ is the friction force, which is usually assumed to be proportional to the velocity of the cantilever motion. Hence, this force tends to zero when $\frac{\partial X}{\partial t} \rightarrow 0$. The experiments in [1] were performed for the cantilever frequency $\omega_0 = (k/m)^{1/2} \approx 24 \times 10^3 s^{-1}$ and vibration amplitudes of the order of $10nm$ or less. This corresponds to the characteristic value of the velocity equal to $0.02cm/s$. This slow cantilever motion, in practice, excludes dissipation mechanisms due

to both Joule losses and fluctuations of the van der Waals forces. Therefore, it seems to be quite reasonable to consider a velocity-independent friction force, F , as the one responsible for dissipation in this case. The conventional term “velocity-independent force” means a force which does not depend on the absolute value of the velocity but always acts in the direction opposite to the tip velocity. Thus, F reverses its sign after each half-period of vibrations. This force occurs in mechanical systems in which one solid surface is in contact with another and slides along it.

The friction force is proportional to the effective contact area, which, depends linearly on the compressing force (the Coulomb friction law). A very exotic mechanism of velocity-independent friction due to photon exchange between surfaces which are in close proximity to one another (but not in a contact) was predicted in Ref. [17].

For the case of a cantilever [1] whose body has a complex multilayer structure, an internal friction force between the different layers may arise. The body is a platinum-coated single-crystal silicon beam. There is also a $1nm$ titanium layer between the $10nm$ platinum “coat” and the silicon. Besides that, the tip was coated with $200nm$ of gold by evaporation. It was established experimentally that the dissipation of the platinum-coated single-crystal silicon cantilever is higher by a factor of 6 than that of an uncoated cantilever. Therefore, a multi-layer structure is essential in forming the overall dissipation. It seems to be reasonable to consider a tip-sample stretching force to be responsible for increasing the contact interlayer area and, consequently, the frictional force, both of which linearly depend on the tip-sample attraction.

The cantilever motion is described by the equation

$$m \frac{\partial^2 X}{\partial t^2} + kX = F, \quad (2)$$

where $F > 0$ for $\frac{\partial X}{\partial t} < 0$ and $F < 0$ for $\frac{\partial X}{\partial t} > 0$. Formally, Eq. 2 may be rewritten in the form given by Eq. 1, with the coefficient Γ depending on the velocity:

$$\Gamma = |F| \left| \frac{\partial X}{\partial t} \right|^{-1}. \quad (3)$$

The oscillator equation with Γ given by Eq. 3 is no longer a linear equation. Nevertheless, it can be easily solved for each time interval between any two

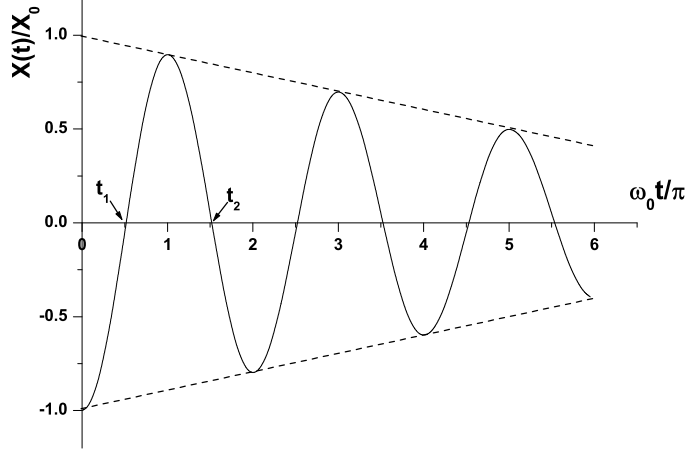


Figure 1: Ringdown oscillations in the case of a velocity-independent force. Points, where $X = 0$, are slightly displaced from the values $n + \frac{1}{2}$ which would correspond to harmonic oscillations.

successive turning points. The general solution is given by

$$X(t) = (-1)^{n+1} \frac{|F|}{k} + [-X_0 + \frac{|F|}{k}(2n+1)] \cos(\omega_0 t), \quad (4)$$

where n is the integer part of $\omega_0 t / \pi$ and $-X_0 < 0$ is the initial position of the tip. Eq. 4 describes the decaying motion of the cantilever. In the range of small dissipation (the energy loss per period is small compared with the total energy ε) and small values of time, Eq. 4 may be approximated by a simplified formula

$$X(t) \approx -X_0(1 - \alpha t) \cos(\omega_0 t) \approx -X_0 e^{-\alpha t} \cos(\omega_0 t), \quad (5)$$

where

$$\alpha \equiv \alpha(X_0) = \frac{2|F|}{\pi m \omega_0 X_0}. \quad (6)$$

The coefficient α describes the nonlinear damping of the oscillations, and it depends strongly on the amplitude X_0 . As we see, the nonlinearity is the most pronounced for small amplitudes as was observed experimentally [1] (at amplitudes of the order of or less than $10nm$). It was noticed in Ref. [1] that

this effect vanishes if either the amplitude of oscillation or the tip-sample separation increases.

With the known oscillator trajectory described by Eq. (4), it is easy to estimate the value of nonlinear shift of the oscillation frequency. In what follows we consider the so called “ringdown” regime of oscillations, which takes place when a drive circuit is abruptly grounded and the cantilever rings down until thermal equilibrium is established. The value of the cantilever frequency may be obtained by measuring the time interval $t_2 - t_1$ between two successive crossing points where the displacement X is equal to zero. For our model, these points do not match with the values $\pi/2\omega_0$ and $3\pi/2\omega_0$ as would occur for harmonic oscillations (see Fig. 1), but are slightly displaced to greater values of t . Considering these displacements as small quantities, we can easily obtain the difference $t_2 - t_1$.

Let us introduce the notation, $\omega_0 t_1 \equiv \frac{\pi}{2} + \delta_1$ and $\omega_0 t_2 \equiv \frac{3\pi}{2} + \delta_2$. Then we obtain from Eq. (4) two equations for δ_1 and δ_2 . They are

$$\begin{aligned} \frac{|F|}{k} + \left(-X_0 + \frac{|F|}{k} \right) \sin \delta_1 &= 0 \text{ and} \\ \frac{|F|}{k} + \left(-X_0 + 3\frac{|F|}{k} \right) \sin \delta_2 &= 0. \end{aligned} \quad (7)$$

It follows from Eqs. (7) that $\delta_1 \approx \frac{|F|}{kX_0} \left(1 + \frac{|F|}{kX_0} \right)$ and $\delta_2 \approx \frac{|F|}{kX_0} \left(1 + 3\frac{|F|}{kX_0} \right)$. Then we have

$$t_2 - t_1 = \frac{\pi}{\omega_0} + \frac{2F^2}{\omega_0 k^2 X_0^2}. \quad (8)$$

The corresponding value of the frequency is given by

$$\omega \approx \frac{\pi}{t_2 - t_1} \approx \omega_0 \left(1 - \frac{2F^2}{\pi k^2 X_0^2} \right) = \omega_0 \left(1 - \frac{F^2}{\pi k \varepsilon} \right), \quad (9)$$

where the energy of vibrations, ε , is equal to $kX_0^2/2$.

The experimentally observed decrease of the frequency is illustrated in Fig. 2d of reference [1]. The decrease in the oscillation frequency takes place at the beginning of the transition from high- to small-amplitude regimes of oscillations. It would be interesting to study experimentally the dependence of the frequency shift on the tip-sample attraction, F_{attr} . For the case of the Coulomb law for the friction force, i.e. when $|F| = c_0 + c_1 F_{attr}$ ($c_{0,1}$ are

constants), the frequency shift is given by the expression

$$\Delta\omega \approx -\frac{\omega_0}{\pi k\varepsilon} \left(c_0 + c_1 F_{attr} \right)^2. \quad (10)$$

When the attraction force is induced by the bias voltage, V , $F_{attr} \sim V^2$. When it is due to localized charges in the dielectric sample, $F_{attr} \sim n_{ch}$, where n_{ch} is the concentration of charges. In both cases the model proposed here can be examined experimentally.

There is an alternative method of calculation of the frequency shift. It is based on the asymptotic theory of nonlinear oscillations developed by Bogolubov and Mitropolskii in [18]. The direct application of this method to our specific case of a velocity-independent friction force results in a frequency shift which is slightly smaller (by factor of $3/\pi$) than that given by Eq. 9. The details are in Appendix A.

So far our analysis assumed a constant friction force. This force may be considered as originating from many discrete events of energy losses. For brevity, we will use the term “kick” for each such event. The duration of each kick is assumed to be very short compared with the period of vibrations, and the number of kicks per period is large. After averaging over a time interval much shorter than the period of oscillations, but large enough to include many kicks, the effect of kicks reduces to a constant friction force. It is evident that the description based on a continuous friction force is applicable to high-amplitude oscillations only. The discrete nature of the dissipation may reveal itself at small amplitudes when the mean “free path” l is longer than or of the same order as the amplitude. The correspondence of the two models to each other imposes a relation between the parameters of both $|F| = \Delta\epsilon/l$, where $\Delta\epsilon$ is the energy loss in the course of each kick ($\Delta\epsilon \ll \varepsilon$).

In what follows, we consider the case of a low-probability kick per oscillation period. This is possible when $X_0 \ll l$. If the kick occurs during the motion away from (towards) the center, the time of returning to the central position ($X = 0$) is decreased (increased). Hence, the period of oscillations is a fluctuating quantity. It can be shown that these fluctuations result in a positive frequency shift that is in contrast to the case of large amplitudes, $X_0 \gg l$. The following simple analysis makes it possible to obtain the value of the shift in an explicit form. As before, we proceed from the equation of motion for a harmonic oscillator

$$X(t) = X_0 \cos(\omega_0 t), \quad (11)$$

which describes the vibrations in the absence of dissipation. Initially, the oscillator is in the position $X(t) = X_0$. Let us assume that the kick occurs at time t_1 . Then $X(t_1) \equiv X_1 = X_0 \cos(\omega_0 t_1)$. The value of t_1 can be expressed via X_1 as

$$t_1 = \frac{1}{\omega_0} \cos^{-1} \left(\frac{X_1}{X_0} \right).$$

After this event, the oscillator continues its motion with a modified amplitude C , which can be obtained from energy conservation. It is given by

$$C = \sqrt{X_0^2 - 2\Delta\epsilon/k}. \quad (12)$$

For simplicity, we will consider that $\Delta\epsilon \ll \epsilon$.

The modified equation of motion is given by

$$X(t) = C \cos(\omega_0 t + \varphi), \quad (13)$$

where φ is the phase variation introduced by the kick. It is convenient to introduce the new time variable, $\tau = t + \varphi/\omega_0$. It can be easily seen that τ varies from $\frac{1}{\omega_0} \cos^{-1} \frac{X_1}{C}$ (the instant of kick) to $\frac{\pi}{2\omega_0}$ (the instant of crossing the x axes). Then the overall time required to get to the central position $X = 0$ is given by

$$t_1 + \frac{\pi}{2\omega_0} - \frac{1}{\omega_0} \cos^{-1} \frac{X_1}{C} \equiv \frac{\pi}{2\omega_0} + \tau^+,$$

where

$$\tau^+ = \frac{1}{\omega_0} \left(\cos^{-1} \frac{X_1}{X_0} - \cos^{-1} \frac{X_1}{C} \right). \quad (14)$$

It follows from Eq. 14 that the time required to reach the center has increased due to the kick. In contrast, the transit time decreases when the oscillator moves away from the center. The corresponding time is given by $\frac{\pi}{2\omega_0} + \tau^-$, where

$$\tau^- = \frac{1}{\omega_0} \left(\sin^{-1} \frac{X_1}{X_0} - \sin^{-1} \frac{X_1}{C} \right). \quad (15)$$

As we see the variation of the transit time depends essentially on the position, X_1 , of the oscillator at the instant of kick. We assume that all realizations of X_1 have equal probabilities. This assumption will make it possible to take into account the contribution of the transit-time fluctuations to the frequency shift.

As before, our following analysis concerns the case of the ringdown experiment. The time required to undertake N full cycles can be expressed as

$$t = N \frac{2\pi}{\omega_0} (1 + \Delta), \quad (16)$$

where

$$\Delta = \frac{1}{2\pi N} \left[\sum_i \tau^-(X_i) n_i + \sum_j \tau^+(X_j) n_j \right].$$

The indices i and j denote the quarter-periods with away-from-center and towards-center motion, respectively. The random variables $n_{i,j}$ are equal to 1 or 0 depending on whether or not a kick occurs during the (i, j) th quarter-period. The average value of n_i is given by $\langle n_i \rangle = \frac{X_0}{t} \ll 1$, implying a negligible probability of two kicks during any given quarter-period. The period of oscillations, T , is defined as $T = t/N$. Hence, the frequency of oscillations is given by

$$\omega = 2\pi N/t = \omega_0 (1 + \Delta)^{-1} \approx \omega_0 (1 - \Delta + \Delta^2 - \dots). \quad (17)$$

After statistical averaging of ω , the linear term $\langle \Delta \rangle$ vanishes. This follows from the definitions of τ^\pm and the trigonometry relation $\sin^{-1}(x) = \frac{\pi}{2} - \cos^{-1}(x)$. The main contribution to the frequency shift is from the term $\langle \Delta^2 \rangle$. Thus, we have

$$\langle \omega \rangle - \omega_0 \approx \frac{\omega_0 \langle n \rangle}{\pi^2 N} \langle [\tau^-(X_i)]^2 \rangle. \quad (18)$$

In deriving Eq. 20, we use the following relations: (i) $n_i^2 \equiv n_i$, (ii) $\langle n_i n_j \rangle = \langle n_i \rangle \langle n_j \rangle$ when $i \neq j$. Also, the integer N cannot be very large because the energy dissipation during the observation time is assumed to be small.

Considering the losses as statistically independent events which have equal probabilities at any value of X within the interval $[-C, +C]$ (the interval where a kick is possible), we can calculate the average value of $[\tau^-]^2$ as

$$\langle [\tau^-]^2 \rangle = \frac{1}{C} \int_0^C dX \left[\sin^{-1}\left(\frac{X}{C}\right) - \sin^{-1}\left(\frac{X}{X_0}\right) \right]^2. \quad (19)$$

This integral can be calculated analytically considering $\frac{\Delta \varepsilon}{2\varepsilon} \ll 1$. (See Appendix B). Finally, we get

$$\langle \omega \rangle - \omega_0 \approx \frac{|F| \Delta \varepsilon}{4\sqrt{2}\pi^2 N \sqrt{m\varepsilon^{3/2}}} \ln\left(\frac{2\varepsilon}{\Delta \varepsilon}\right). \quad (20)$$

Comparing the amplitude dependence of the frequency shift in two limiting cases given by Eqs. 9 and 20, we may conclude that the function $\omega(X_0)$ has a minimum at intermediate amplitudes as shown in Fig. 2. This curve

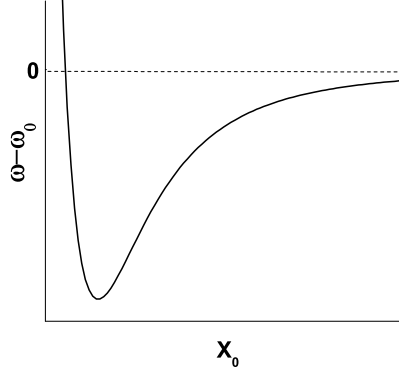


Figure 2: Dependence of frequency shift on the amplitude of oscillations X_0 . is similar to the experimental one displayed in Fig. 2d of reference [1].

3 Conclusion

We have described phenomenologically a possible mechanism that produces a small-amplitude nonlinearity for cantilever vibrations. It is assumed that the nonlinearity is due to velocity-independent friction force which, at small amplitudes, reduces to a sequence of random kicks retarding cantilever motion. Our analysis predicts both the increase of the decrement of oscillations and nonmonotonic behavior of the frequency shift when the amplitude decreases. It is shown that the last effect is due to the crossover from the amplitude-independent friction force to discrete events of the cantilever energy losses. This is in a qualitative agreement with the cantilever vibrations observed experimentally.

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Appendix A

A regular method for solution of the equations

$$\ddot{X}(t) + \omega_0^2 X = \epsilon f(X, \dot{X}), \quad (21)$$

where ϵ is a dimensionless quantity ($\epsilon \rightarrow 0$), is developed in [18]. In our case, the term on the right-hand side is given by

$$\epsilon f(X, \dot{X}) \equiv -\frac{F}{m} \frac{\dot{X}}{|\dot{X}|}. \quad (22)$$

According to [18], the solution of Eq. 21 can be represented as an expansion in powers of ϵ ,

$$\begin{aligned} X(t) &= a \cos \varphi + \epsilon u_1(a, \varphi) + \epsilon^2 u_2(a, \varphi) + O(\epsilon^3), \\ \dot{a} &= \epsilon A_1(a) + \epsilon^2 A_2(a) + O(\epsilon^3), \\ \dot{\varphi} &= \omega_0 + \epsilon B_1(a) + \epsilon^2 B_2(a) + O(\epsilon^3). \end{aligned} \quad (23)$$

To obtain the decrement and frequency shift of oscillations, it is sufficient to calculate only terms up to the second order in ϵ^2 . For the case of the dissipation force given by Eq. 22, these quantities are determined by

$$\begin{aligned} A_1(a) &= \frac{F_1}{2\omega_0}, \\ B_1(a) &= A_2(a) = 0, \\ B_2(a) &= -\frac{F_1^2}{4\omega_0^3 a^2} - \frac{1}{2\omega_0^3 a^2} \sum_{n=2}^{\infty} \frac{n^2 F_n^2}{n^2 - 1}, \end{aligned} \quad (24)$$

where

$$F_n = -\frac{4|f|}{\pi n} \sin\left(\frac{\pi n}{2}\right)$$

The quantity A_1 determines the decay of oscillations. Setting the initial condition for the amplitude a corresponding to the solution given by Eq. 4, $a(t=0) = -X_0$, we obtain from Eqs. 23 and 24

$$a(t) = -X_0 \left(1 - \frac{2Ft}{\pi m \omega_0 X_0} \right). \quad (25)$$

This coincides with the results given by Eqs. 5 and 6.

The value of the renormalized frequency $\omega(a) = \dot{\varphi}(a)$ can be obtained explicitly after summing up in the expression for $B_2(a)$. This sum is equal to $F_1^2/4$. Then the frequency of oscillations is given by

$$\omega = \omega_0 \left(1 - \frac{6F^2}{\pi^2 k^2 X_0^2} \right). \quad (26)$$

The second term in the brackets is very close to that in Eq. 9. Their ratio is equal to $3/\pi$ which is close to unity. Thus both approaches give almost identical results.

Appendix B

$$\begin{aligned} \langle [\tau^-]^2 \rangle &= \frac{1}{C} \int_0^C dX \left[\sin^{-1} \left(\frac{X}{C} \right) - \sin^{-1} \left(\frac{X}{X_0} \right) \right]^2 = \\ &= \int_0^1 dx [\sin^{-1} x - \sin^{-1} x(1-\delta)]^2, \end{aligned} \quad (27)$$

where $\delta \equiv \Delta\varepsilon/(2\varepsilon) \ll 1$. Let us set

$$\sin^{-1} x(1-\delta) = y + \delta',$$

where $|\delta'| \ll 1$. Then we have $\sin y = x$ and

$$\delta' \approx \sqrt{x^{-2} - 1} - \sqrt{x^{-2} - 1 + 2\delta}.$$

After substituting $x = (1+z)^{-1/2}$, the last integral in (27) is reduced to

$$\delta^2 \int_0^\infty \frac{dz}{(1+z)^{3/2}} \frac{1}{z + \delta + \sqrt{z^2 + 2z\delta}}. \quad (28)$$

One cannot set $\delta = 0$ in the integrand of Eq. (28) because of logarithmic divergence of the integral.

Let us denote the last integral as K . It is convenient to divide the range of integration in two parts and express the whole integral as a sum $K = K_1 + K_2$, where

$$K_1 = \int_0^{\sqrt{\delta}} ; K_2 = \int_{\sqrt{\delta}}^{\infty} .$$

The integrand in K_1 can be approximated by $[z + \delta + \sqrt{z^2 + 2z\delta}]^{-1}$, and it can be easily seen that $K_1 \approx (1/4)|\ln\delta|$.

The integrand in K_2 can be approximated by $\left[(1+z)^{3/2}2z\right]^{-1}$. For small values of δ , $K_2 \approx (1/4)|\ln\delta|$. Then, $K \approx (1/2)|\ln\delta|$. Finally, using Eqs. (27) and (28), we have

$$\langle[\tau^-]^2\rangle = \frac{\Delta\varepsilon^2}{8\varepsilon^2} \ln \frac{2\varepsilon}{\Delta\varepsilon}. \quad (29)$$

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